Episode 6.01 - Introduction to Karnaugh Maps

(Transcript URL: https://intermation.com/episode-6-01-introduction-to-karnaugh-maps/)

Show Description: Here we introduce a graphical tool that when used correctly will produce a most simplified sum-of-products expression, all without meddling in any Boolean expression simplification.

<table>
<tr><th>Podcast Timestamp</th><th>Supporting Details</th></tr>
<tr><td>1:20</td><td>
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>X</th><th></th></tr>
<tr><td><i>1st row →</i></td><td>0</td><td>0</td><td>0</td><td>0</td><td></td></tr>
<tr><td><i>2nd row →</i></td><td>0</td><td>0</td><td>1</td><td>0</td><td></td></tr>
<tr><td><i>3rd row →</i></td><td>0</td><td>1</td><td>0</td><td>1</td><td>$\leftarrow \overline{A}\cdot B\cdot \overline{C}$</td></tr>
<tr><td><i>4th row →</i></td><td>0</td><td>1</td><td>1</td><td>0</td><td></td></tr>
<tr><td><i>5th row →</i></td><td>1</td><td>0</td><td>0</td><td>1</td><td>$\leftarrow A\cdot \overline{B}\cdot \overline{C}$</td></tr>
<tr><td><i>6th row →</i></td><td>1</td><td>0</td><td>1</td><td>0</td><td></td></tr>
<tr><td><i>7th row →</i></td><td>1</td><td>1</td><td>0</td><td>1</td><td>$\leftarrow A\cdot B\cdot \overline{C}$</td></tr>
<tr><td><i>8th row →</i></td><td>1</td><td>1</td><td>1</td><td>1</td><td>$\leftarrow A\cdot B\cdot C$</td></tr>
</table>
Result: $X=\overline{A}\cdot B\cdot \overline{C}+A\cdot \overline{B}\cdot \overline{C}+A\cdot B\cdot \overline{C}+A\cdot B\cdot C$
</td></tr>
<tr><td>3:18</td><td>
Simplification of third and seventh rows:
<table>
<tr><td>$\overline{A}\cdot B\cdot \overline{C}+A\cdot B\cdot \overline{C}$</td><td>This expression is not in SOP form</td></tr>
<tr><td>$B\cdot \overline{C}\cdot(\overline{A}+A)$</td><td>By distributive law, pull out $B\cdot \overline{C}$</td></tr>
<tr><td>$B\cdot \overline{C}\cdot 1$</td><td>By inverse law for AND, $\overline{A}+A=1$</td></tr>
<tr><td>$B\cdot \overline{C}$</td><td>By identity law, anything AND-ed with1 is itself</td></tr>
</table>
</td></tr>
<tr><td>3:37</td><td>
Simplification of fifth and seventh rows:
<table>
<tr><td>$A\cdot \overline{B}\cdot \overline{C}+A\cdot B\cdot \overline{C}$</td><td>This expression is not in SOP form</td></tr>
<tr><td>$A\cdot \overline{C}\cdot(\overline{B}+B)$</td><td>By distributive law, pull out $A\cdot \overline{C}$</td></tr>
<tr><td>$A\cdot \overline{C}\cdot 1$</td><td>By inverse law for AND, $\overline{B}+B=1$</td></tr>
<tr><td>$A\cdot \overline{C}$</td><td>By identity law, anything AND-ed with1 is itself</td></tr>
</table>
</td></tr>
</table>

Podcast Timestamp | Supporting Details

3:52

Simplification of seventh and eighth rows:

$A \cdot B \cdot \overline{C} + A \cdot B \cdot C$ This expression is not in SOP form

$A \cdot B \cdot (\overline{C} + C)$ By distributive law, pull out $A \cdot B$

$A \cdot B \cdot 1$ By inverse law for AND, $\overline{C} + C = 1$

$A \cdot B$ By identity law, anything AND-ed with1 is itself

4:10

Simplification of full SOP expression derived from truth table

$$X = \overline{A} \cdot B \cdot \overline{C} + A \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$$

$$X = \overline{A} \cdot B \cdot \overline{C} + \overset{\textit{(added)}}{A \cdot B \cdot \overline{C}} + A \cdot \overline{B} \cdot \overline{C} + \overset{\textit{(added)}}{A \cdot B \cdot \overline{C}} + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$$

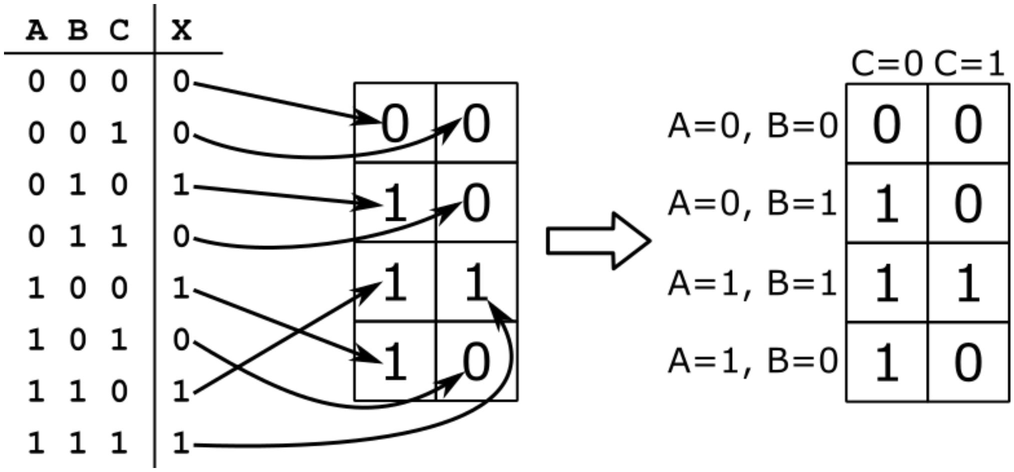
$$X = B \cdot \overline{C} \cdot (\overline{A} + A) + A \cdot \overline{C} \cdot (\overline{B} + B) + A \cdot B \cdot (\overline{C} + C)$$

$$X = B \cdot \overline{C} \cdot 1 + A \cdot \overline{C} \cdot 1 + A \cdot B \cdot 1$$

$$X = B \cdot \overline{C} + A \cdot \overline{C} + A \cdot B$$

6:11

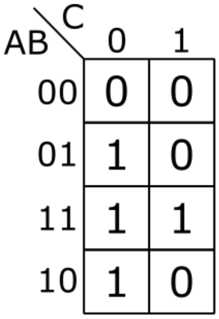
Mapping of truth table to Karnaugh map

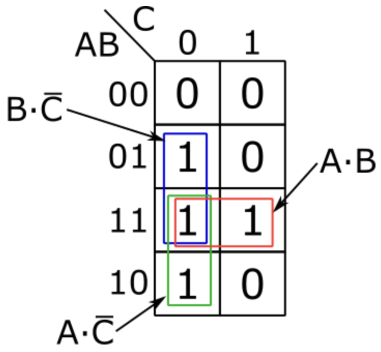


A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

	C=0	C=1
A=0, B=0	0	0
A=0, B=1	1	0
A=1, B=1	1	1
A=1, B=0	1	0

Podcast Timestamp	Supporting Details
12:08	Karnaugh map with conventional notation





Sample Problems

1. Transfer the output values from each of the truth tables shown below to 3-input Karnaugh maps.

A	*B*	*C*	*X*
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

A	*B*	*C*	*X*
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

A	*B*	*C*	*X*
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

2. For each of the 3-input Karnaugh maps shown below, draw a rectangle around each pair of ones that will simplify to a single product.* After identifying all of the pairs, create each product, then combine them into the final sum-of-products expression.

AB \ C	0	1
00	1	0
01	1	0
11	0	1
10	0	1

AB \ C	0	1
00	0	1
01	0	0
11	1	1
10	0	1

AB \ C	0	1
00	0	1
01	0	1
11	1	1
10	1	0

* - Note that in later episodes, we will address Karnaugh maps where the ones cannot be paired, where redundant pairs can be eliminated, and where ones should be combined in groups larger than two. For this problem, simply identify the pairs.