

## Episode 5.03 - The Product-of-Sums Expression

(Transcript URL: <https://intertainment.com/episode-5-03-the-product-of-sums-expression/>)

**Show Description:** Now that we've studied the sum-of-products form of Boolean expressions, it's time to take a look at the product-of-sums. This form uses a logical OR to generate zeros which are passed to the output through an AND gate.

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	<p>Example SOP expression: <math>X = (\bar{A} + B + C + \bar{D}) \cdot (A + \bar{B} + D) \cdot (A + \bar{D})</math></p> <table border="1"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>X</th> <th></th> </tr> </thead> <tbody> <tr> <td>1st row →</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td></td> </tr> <tr> <td>2nd row →</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>← <math>A + \bar{D}</math> (1 of 4 zeros)</td> </tr> <tr> <td>3rd row →</td> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td></td> </tr> <tr> <td>4th row →</td> <td>0</td> <td>0</td> <td>1</td> <td>1</td> <td>0</td> <td>← <math>A + \bar{D}</math> (2 of 4 zeros)</td> </tr> <tr> <td>5th row →</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>← <math>A + \bar{B} + D</math> (1 of 2 zeros)</td> </tr> <tr> <td>6th row →</td> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td>0</td> <td>← <math>A + \bar{D}</math> (3 of 4 zeros)</td> </tr> <tr> <td>7th row →</td> <td>0</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> <td>← <math>A + \bar{B} + D</math> (2 of 2 zeros)</td> </tr> <tr> <td>8th row →</td> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> <td>← <math>A + \bar{D}</math> (4 of 4 zeros)</td> </tr> <tr> <td>9th row →</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td></td> </tr> <tr> <td>10th row →</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>← <math>\bar{A} + B + C + \bar{D}</math> (1 of 1 zeros)</td> </tr> <tr> <td>11th row →</td> <td>1</td> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td></td> </tr> <tr> <td>12th row →</td> <td>1</td> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td></td> </tr> <tr> <td>13th row →</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> <td></td> </tr> <tr> <td>14th row →</td> <td>1</td> <td>1</td> <td>0</td> <td>1</td> <td>1</td> <td></td> </tr> <tr> <td>15th row →</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> <td>1</td> <td></td> </tr> <tr> <td>16th row →</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td></td> </tr> </tbody> </table>		A	B	C	D	X		1st row →	0	0	0	0	1		2nd row →	0	0	0	1	0	← $A + \bar{D}$ (1 of 4 zeros)	3rd row →	0	0	1	0	1		4th row →	0	0	1	1	0	← $A + \bar{D}$ (2 of 4 zeros)	5th row →	0	1	0	0	0	← $A + \bar{B} + D$ (1 of 2 zeros)	6th row →	0	1	0	1	0	← $A + \bar{D}$ (3 of 4 zeros)	7th row →	0	1	1	0	0	← $A + \bar{B} + D$ (2 of 2 zeros)	8th row →	0	1	1	1	0	← $A + \bar{D}$ (4 of 4 zeros)	9th row →	1	0	0	0	1		10th row →	1	0	0	1	0	← $\bar{A} + B + C + \bar{D}$ (1 of 1 zeros)	11th row →	1	0	1	0	1		12th row →	1	0	1	1	1		13th row →	1	1	0	0	1		14th row →	1	1	0	1	1		15th row →	1	1	1	0	1		16th row →	1	1	1	1	1	
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Podcast Timestamp	Supporting Details						
<b>6:41</b>	<i>A</i>	<i>B</i>	<i>C</i>	$A+B+C$	$\bar{A}+B+C$	$\bar{A}+\bar{B}+C$	$(A+B+C) \cdot (\bar{A}+B+C) \cdot (\bar{A}+\bar{B}+C)$
	0	0	0	0	1	1	0
	0	0	1	1	1	1	1
	0	1	0	1	1	1	1
	0	1	1	1	1	1	1
	1	0	0	1	0	1	0
	1	0	1	1	1	1	1
	1	1	0	1	1	0	0
	1	1	1	1	1	1	1

<b>9:07</b>	<i>A</i>	<i>B</i>	<i>C</i>	$A+B+C$	$A+\bar{B}+C$	$\bar{A}+B+\bar{C}$	$\bar{A}+\bar{B}+\bar{C}$	$(A+B+C) \cdot (A+\bar{B}+C) \cdot (\bar{A}+B+\bar{C}) \cdot (\bar{A}+\bar{B}+\bar{C})$
	0	0	0	0	1	1	1	0
	0	0	1	1	1	1	1	1
	0	1	0	1	0	1	1	0
	0	1	1	1	1	1	1	1
	1	0	0	1	1	1	1	1
	1	0	1	1	1	0	1	0
	1	1	0	1	1	1	1	1
	1	1	1	1	1	1	0	0

**Sample Problems**

- Derive the truth table for each of the following three-input POS expressions.
  - $(\bar{A}+B+\bar{C}) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+\bar{B}+C)$
  - $(\bar{A}+\bar{B}+\bar{C}) \cdot (B+\bar{C})$
  - $(B+\bar{C}) \cdot (\bar{A}+\bar{C}) \cdot (A+\bar{B}+C)$
- How many zeros does the sum  $(\bar{A}+\bar{E})$  generate in a truth table with 5 input variables?

3. Derive the POS expression for each of the 3-input truth tables shown below.

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1