

Episode 4.10 - More Boolean Simplifications

(Transcript URL: <https://intertainment.com/episode-4-10-more-boolean-simplifications/>)

Show Description: Because many students experience difficulties when trying to simplify Boolean expressions, we're going to dedicate another episode to examples of simplification. We're also going to show how sometimes, there's more than one way to crack an egg.

Podcast Timestamp	Supporting Details
1:24	<p>Prove $\overline{A \cdot B \cdot C} + B \cdot C = 1$ (Method 1)</p> $\overline{A \cdot B \cdot C} + B \cdot C \quad 0.) \text{ Given}$ $\overline{A \cdot (B \cdot C)} + B \cdot C \quad 1.) \text{ Associative law: group } B \cdot C \text{ under the bar}$ $\overline{A} + \overline{(B \cdot C)} + B \cdot C \quad 2.) \text{ DeMorgan: distribute bar to } \overline{A} \text{ \& } \overline{(B \cdot C)}$ $\overline{A} + (\overline{(B \cdot C)} + (B \cdot C)) \quad 3.) \text{ Associative law: group } \overline{(B \cdot C)} + B \cdot C$ $\overline{A} + 1 \quad 4.) \text{ Inverse law: replace } \overline{(B \cdot C)} + B \cdot C \text{ with } 1$ $1 \quad 5.) \text{ Annulment law: anything OR-ed with } 1 \text{ is } 1$
4:36	$A + \overline{A} \cdot B = A + B$
4:03	<p>Prove $\overline{A \cdot B \cdot C} + B \cdot C = 1$ (Method 2)</p> $\overline{A \cdot B \cdot C} + B \cdot C \quad 0.) \text{ Given}$ $\overline{A} + \overline{B} + \overline{C} + B \cdot C \quad 1.) \text{ DeMorgan.: distribute bar to } \overline{A}, \overline{B}, \text{ \& } \overline{C}$ $\overline{A} + \overline{B} + (\overline{C} + B) \cdot (\overline{C} + C) \quad 2.) \text{ Distribute } \overline{C} \text{ OR'ed across product } B \cdot C$ $\overline{A} + \overline{B} + (\overline{C} + B) \cdot 1 \quad 3.) \text{ Inverse law: replace } \overline{C} + C \text{ with } 1$ $\overline{A} + \overline{B} + (\overline{C} + B) \quad 4.) \text{ Identity law: anything AND'ed w/1 is itself}$ $\overline{A} + \overline{B} + \overline{C} + B \quad 5.) \text{ Associative law: lose parenthesis}$ $\overline{A} + \overline{B} + B + \overline{C} \quad 6.) \text{ Commutative law: swap } \overline{C} \text{ and } B$ $\overline{A} + 1 + \overline{C} \quad 7.) \text{ Inverse law: } \overline{B} + B = 1$ $1 \quad 8.) \text{ Annulment law: anything OR-ed with } 1 \text{ is } 1$

Podcast Timestamp	Supporting Details
6:52	<p>Prove $(A \cdot B + A \cdot B \cdot C \cdot D) \cdot (\bar{A} + \bar{B}) = 0$ (Method 1)</p> <p>$(A \cdot B + A \cdot B \cdot C \cdot D) \cdot (\bar{A} + \bar{B})$ 0.) Given</p> <p>$A \cdot B \cdot \bar{A} + A \cdot B \cdot \bar{B} + A \cdot B \cdot C \cdot D \cdot \bar{A} + A \cdot B \cdot C \cdot D \cdot \bar{B}$ 1.) Apply FOIL to factored expression</p> <p>$A \cdot \bar{A} \cdot B + A \cdot B \cdot \bar{B} + A \cdot \bar{A} \cdot B \cdot C \cdot D + A \cdot B \cdot \bar{B} \cdot C \cdot D$ 2.) Commutative law: rearrange product terms so inverses are next to each other</p> <p>$0 \cdot B + A \cdot 0 + 0 \cdot B \cdot C \cdot D + A \cdot 0 \cdot C \cdot D$ 3.) Inverse law: anything AND-ed w/inverse is 0</p> <p>$0 + 0 + 0 + 0$ 4.) Annulment law: anything AND-ed w/0 is 0</p> <p>0 5.) All zeros input to an OR output a zero</p>
9:31	<p>Prove $(A \cdot B + A \cdot B \cdot C \cdot D) \cdot (\bar{A} + \bar{B}) = 0$ (Method 2)</p> <p>$(A \cdot B + A \cdot B \cdot C \cdot D) \cdot (\bar{A} + \bar{B})$ 0.) Given</p> <p>$(A \cdot B + A \cdot B \cdot C \cdot D) \cdot \overline{(A \cdot B)}$ 1.) DeMorgan: $\overline{(A \cdot B)} = \bar{A} \cdot \bar{B}$</p> <p>$(A \cdot B) \cdot \overline{(A \cdot B)} + (A \cdot B \cdot C \cdot D) \cdot \overline{(A \cdot B)}$ 2.) Distribute $\overline{(A \cdot B)}$ across $(A \cdot B + A \cdot B \cdot C \cdot D)$</p> <p>$0 + (A \cdot B \cdot C \cdot D) \cdot \overline{(A \cdot B)}$ 3.) Inverse law: anything AND-ed w/inverse is 0</p> <p>$(A \cdot B \cdot C \cdot D) \cdot \overline{(A \cdot B)}$ 4.) Identity law: anything OR-ed with zero is itself</p> <p>$(A \cdot B) \cdot (C \cdot D) \cdot \overline{(A \cdot B)}$ 5.) Associative law: group A and B together</p> <p>$(A \cdot B) \cdot \overline{(A \cdot B)} \cdot (C \cdot D)$ 6.) Commutative law: put $(A \cdot B)$ & $\overline{(A \cdot B)}$ together</p> <p>$0 \cdot (C \cdot D)$ 7.) Inverse law: anything AND-ed w/inverse is 0</p> <p>0 8.) Annulment law: anything AND-ed w/0 is 0</p>

Sample Problems

For sample problems, please see worksheet for Episode 4.09